Collisions

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The collisions of disks on an air-hockey table are to be analyzed. The disks have approximately equal mass and the air-hockey table ensures a roughly frictionless surface. Completely inelastic and mostly elastic collisions were performed, using various collars around the disks.
The videos of these collisions will be analyzed for conservation of momentum and loss of kinetic energy.

1 Situations to be Analyzed

The following situations were videotaped.

- Two quasi-elastic disks colliding straight on
- Two quasi-elastic disks colliding towards each other with forward movement
- A quasi-elastic disk ramming into another, slightly off center
- Two inelastic disks colliding straight on
- Two inelastic disks colliding towards each other with forward movement
- An inelastic disk ramming into another, slightly off center

In each situation, the mass of the disks are not known, but are known to be identical. It shall be assumed that each disk has its mass concentrated almost entirely at its center.
Each video is to be motion tracked to create numerical data describing the motion of each disk in each scenario.

2 Motion Tracking

For motion tracking purposes, it is assumed that the camera records at a constant framerate of exactly 29.97 frames per second.
Additionally, it is assumed that the exact distance between two particular fixed points in the video is known, namely two white dots that are assumed to be precisely 50 **cm** apart.

The error bars in the following graphs only represent the error resulting from imprecise pixel tracking and the limited resolution of the video data.

Here are the position graphs of the disks with start, end, and collision instants labeled by time of occurrence.

Figure 1: Quasi-Elastic Disks Straight On

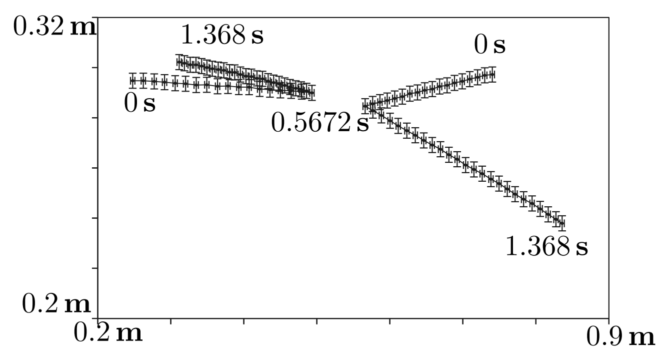


Figure 2: Quasi-Elastic Disks, Forward Motion

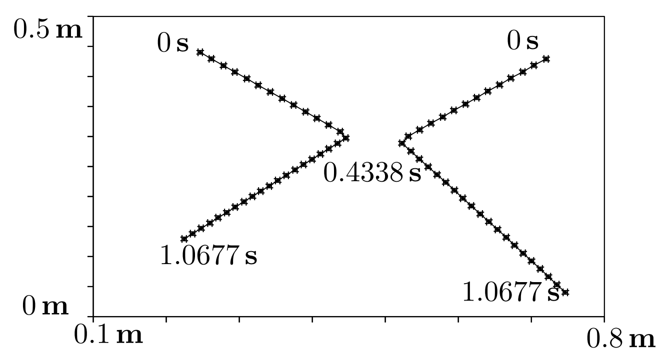
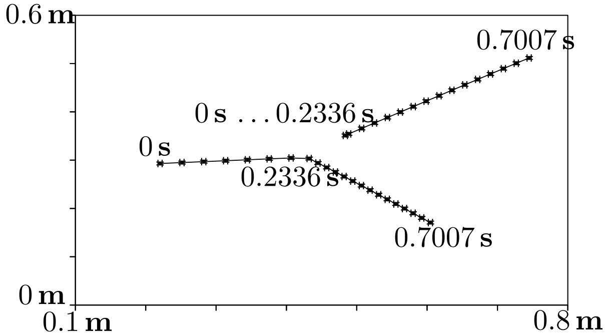


Figure 3: Quasi-Elastic Disks, One Stationary



It can be seen that the straight-on videos were not precisely straight on, resulting in the angle of motion changing after the collision. Furthermore, the inelastic collisions resulted in both objects combining and spinning, resulting in the curved paths seen. The curved paths are difficult to work with. Since the disks are stuck together after the inelastic collision, they act as one rigid body with a fixed center of mass. This center of mass can be tracked. Here are the inelastic graphs with the pair center of mass position overlaid.

Figure 4: Inelastic Disks Straight On

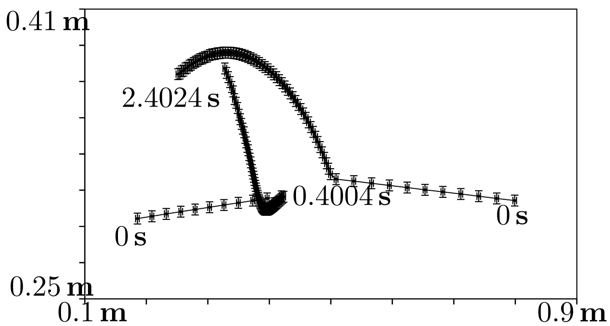


Figure 5: Inelastic Disks, Forward Movement

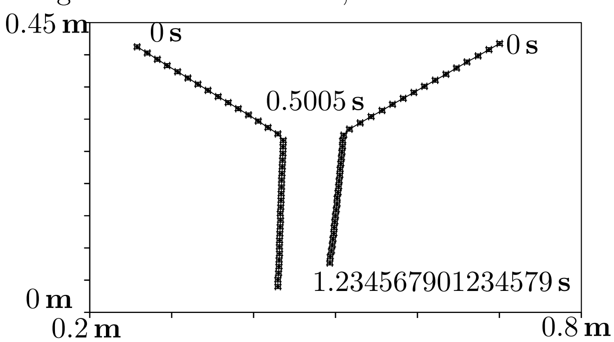


Figure 6: Inelastic Disks, One Stationary

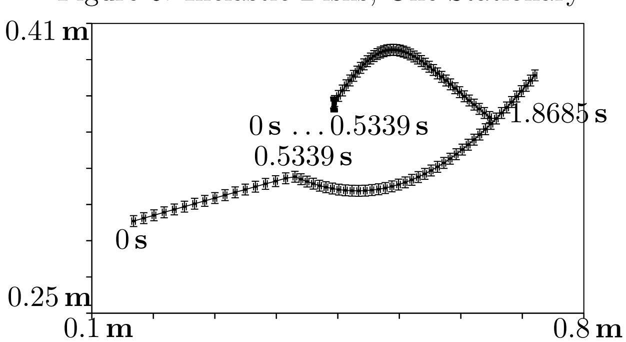


Figure 7: Inelastic, Straight On, Center Of Mass

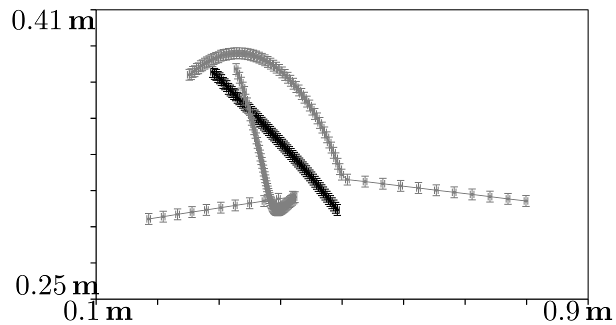


Figure 8: Inelastic, Movement, Center of Mass

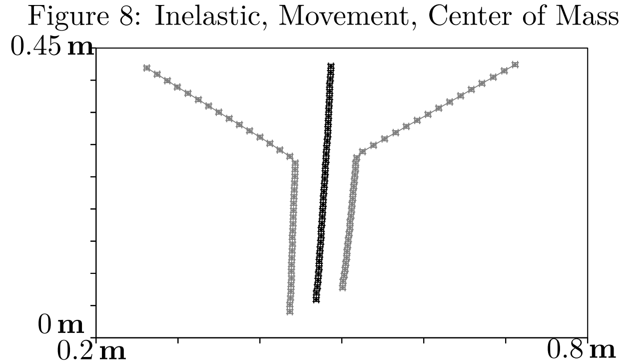
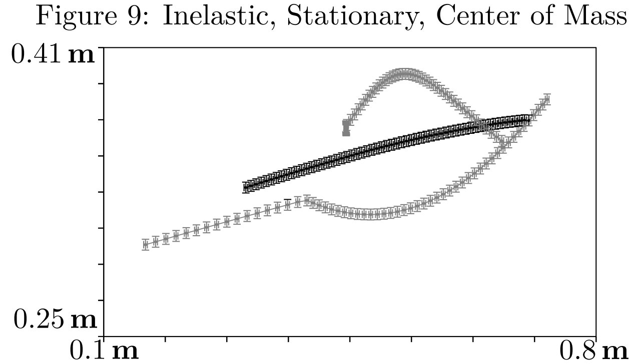


Figure 9: Inelastic, Stationary, Center of Mass



It can be seen that the centers of mass move with constant velocity in the inelastic cases. This is also the case with the elastic disks, as will be demonstrated later.

In Figure 9, the center of mass of the inelastic disks where one began stationary has a clear slight curvature in its path. This and other curvatures is possibly due to the air hockey table being tilted, allowing gravitational forces to influence the motion.

3 Momentum

While Figures 7 through 9 suggest that momentum is conserved, this conservation can be tested for rigorously for both the elastic and inelastic cases. This is done by taking the horizontal and vertical position of the center of mass of the disks over time and running a linear regression to determine the velocity. Since the mass of each disk is identical, the velocity is proportional to the momentum. The mass of a single disk shall be represented by the letter M.

For each situation, each disk, before and after collision, the covariance and variance in time was calculated. The quotient between these two quantities is equal to the slope of the line of least squared residual error, or in this case, the velocity. The intercepts of these linear regressions were found by resolving the regression given the mean values of the data. Using this linear regression, the standard error of the slope was calculated, and a 96.875% confidence interval was constructed the slope of each regression. The uncertainty in position measurements is not considered; only the uncertainty of linear regression matters here.

Here are the results. The Time column indicates whether or not the velocity corresponds to the time before or after the collision. The direction is split into horizontal (H) and vertical (V) components. To reiterate, the center of mass is the point being tracked here.

Table 1: Quasi-Elastic Disks Straight On

Time	Dir.	Momentum	Uncertainty
Before	H	$0.062879M\,\mathrm{m/s}$	$8.30832 \times 10^{-7}M\,\mathrm{m/s}$
After	H	$0.059567M\,\mathrm{m/s}$	$6.40359 \times 10^{-6}M\,\mathrm{m/s}$
Before	V	$-0.015490M\,\mathrm{m/s}$	$5.56734 \times 10^{-7}M\,\mathrm{m/s}$
After	V	$-0.019175M\,\mathrm{m/s}$	$5.81708 \times 10^{-6}M\,\mathrm{m/s}$

Table 2: Quasi-Elastic, Forward Movement

Time	Dir.	Momentum	Uncertainty
Before	H	$0.005198M\,\mathrm{m/s}$	$4.91274 \times 10^{-6}M\,\mathrm{m/s}$
After	H	$0.002720M\,\mathrm{m/s}$	$7.02349 \times 10^{-6}M\,\mathrm{m/s}$
Before	V	$-0.325688M\,\mathrm{m/s}$	$7.82663 \times 10^{-7}M\,\mathrm{m/s}$
After	V	$-0.328060M\,\mathrm{m/s}$	$1.19391 \times 10^{-6}M\,\mathrm{m/s}$

Table 3: Quasi-Elastic Disks, One Stationary

Time	Dir.	Momentum	Uncertainty
Before	H	$0.464686M\,\mathrm{m/s}$	$3.43674 \times 10^{-7}M\,\mathrm{m/s}$
After	H	$0.460838M\,\mathrm{m/s}$	$5.58165 \times 10^{-5}M\,\mathrm{m/s}$
Before	V	$0.029251M\,\mathrm{m/s}$	$3.06074 \times 10^{-8}M\,\mathrm{m/s}$
After	V	$0.027956M\,\mathrm{m/s}$	$4.72994 \times 10^{-5}M\,\mathrm{m/s}$

Table 4: Inelastic Disks Straight On

Time	Dir.	Momentum	Uncertainty
Before	H	$-0.084195M\,\mathrm{m/s}$	$6.16916 \times 10^{-7}M\,\mathrm{m/s}$
After	H	$-0.083874M\,\mathrm{m/s}$	$3.61501 \times 10^{-5}M\,\mathrm{m/s}$
Before	V	$0.036187M\,\mathrm{m/s}$	$1.15337 \times 10^{-7}M\,\mathrm{m/s}$
After	V	$0.032036M\,\mathrm{m/s}$	$1.33556 \times 10^{-4}M\,\mathrm{m/s}$

Table 5: Inelastic, Forward Movement

Time	Dir.	Momentum	Uncertainty
Before	H	$-0.011598M\,\mathrm{m/s}$	$1.79165 \times 10^{-6}M\,\mathrm{m/s}$
After	H	$-0.014507M\,\mathrm{m/s}$	$5.77176 \times 10^{-6}M\,\mathrm{m/s}$
Before	V	$-0.290524M\,\mathrm{m/s}$	$1.30771 \times 10^{-6}M\,\mathrm{m/s}$
After	V	$-0.293249M\,\mathrm{m/s}$	$6.03741 \times 10^{-6}M\,\mathrm{m/s}$

Table 6: Inelastic Disks, One Stationary

Time	Dir.	Momentum	Uncertainty
Before	H	$0.246551M\,\mathrm{m/s}$	$1.66022 \times 10^{-7}M\,\mathrm{m/s}$
After	H	$0.242616M\,\mathrm{m/s}$	$2.76485 \times 10^{-5}M\,\mathrm{m/s}$
Before	V	$0.025778M\,\mathrm{m/s}$	$1.30405 \times 10^{-7}M\,\mathrm{m/s}$
After	V	$0.021174M\,\mathrm{m/s}$	$1.12470 \times 10^{-4}M\,\mathrm{m/s}$

It can be seen that for most cases, the momentum is roughly the same before and after the collision. The difference can be accounted for by the fact that there appears to be some external forces on the system, possibly gravity, friction, or air resistance.

4 Kinetic Energy

While momentum is conserved when there are no external forces present, kinetic energy is not necessarily. An elastic collision is defined as a collision where no kinetic energy is lost; anything less than elastic implies that kinetic energy is lost to heat.

The elasticity of a collision is determined by how much kinetic energy is retained through the collision process. The fraction of the final over initial kinetic energy is the square of what is known as the coefficient of restitution.

The kinetic energy of the system before and after the collision can be determined by summing the kinetic energies of each disk with the equation that equates kinetic energy to $\frac{1}{2}Mv^2$. The velocity of each disk is obtained by linear regression, and the Pythagorean Theorem is used to determine the total velocity based on horizontal and vertical components.

Below are the calculations for initial and final kinetic energy, as well as the fraction of kinetic energy retained (COR^2) of each collision.

Note: the rotational kinetic energy is not put into consideration here. I'd include it if I had more time.

Table 7: Quasi-Elastic Disks, Straight On

Initial K.E.	Final K.E.	COR^2
$0.141343M$ m²/s²	$0.085843M$ m²/s²	0.607337

Table 8: Quasi-Elastic Disks, Forward Motion

Initial K.E.	Final K.E.	COR^2
$0.331717M$ m²/s²	$0.235151M$ m²/s²	0.708892

Table 9: Quasi-Elastic Disks, One Stationary

Initial K.E.	Final K.E.	COR^2
$0.433877M$ m²/s²	$0.373561M$ m²/s²	0.860985

Table 10: Inelastic Disks, Straight On

Initial K.E.	Final K.E.	COR^2
$0.248539M$ m²/s²	$0.008057M$ m²/s²	0.032419

Table 11: Inelastic Disks, Forward Motion

Initial K.E.	Final K.E.	COR^2
$0.233671M$ m²/s²	$0.087222M$ m²/s²	0.373268

Table 12: Inelastic Disks, One Stationary

Initial K.E.	Final K.E.	COR^2
$0.123205M$ m²/s²	$0.058259M$ m²/s²	0.472865

5 Conclusions

The quasi-elastic collisions retain more kinetic energy than the inelastic ones, and head-on collisions result in the most kinetic energy loss. Interestingly, when one disk starts stationary, the kinetic energy is relatively largely conserved, compared to the other two situations. The situation where both disks move forward and collide is perhaps a middle ground between head-on collision and one starting stationary because both particles are still moving toward each other, but there is an extra offset and the collision is not as head on, while still not being as indirect as one particle starting stationary.

Conservation of momentum to some extent constrains the motion of the disks. When the disks collide head on, due to equal mass the sum of their velocities has to stay the same, meaning that the disks can only be moving in exactly opposite directions with equal speed. In the inelastic case, this essentially means that both disks must stop entirely. This contrasts the case when the disks moved forward while having stuck together, still retaining all of the vertical velocity but none of the horizontal velocities, because there was no horizontal momentum to begin with due to the sum equalling 0.