Analysis of Motion

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In this lab, motion of objects on earth is closely analyzed. Software such as Logger Pro is used to extract data on an object's trajectory from sources such as video files. Various models are used and compared for their effectiveness and limitations when describing motion of objects, namely projectiles and pendulums.

1 Introduction

In modern physics, digital devices are used to collect large amounts of data describing physical interactions. While more information is often seen as resulting in more meaningful conclusions, the cost is the effort of sifting through this data to extract only what is needed and to interpret the large amount of data correctly.

As of writing, over a year ago was when the first image of a black hole was constructed. An enormous amount of data was collected using sensors and computers for storage, but the data itself was not enough to trivially produce an image of what it would look like to the human eye. It took extensive research and use of advanced physical models to interpret the data in order to produce that viral first image of a black hole. Even this first interpretation had room for improvement; months after the first image was released, scientists reinterpreted the same data to produce an even more accurate depiction of the same black hole. The interpretation relied heavily on the model of the physics at hand. If the model was incorrect, so would have been the final image.

This lab covers analysis of interactions more earthly than black holes. Videos of a hammer being thrown and a toy skydiver being dropped are observed, as well as numerical data from swinging pendulums. The data is processed and analyzed using simplified Newtonian models in order to make meaningful observations about the objects and the environment.

2 Raw Data

Video files are rich in information, but a video of an object in motion does not explicitly provide data about the motion itself. Motion tracking software was used to trace the location of objects shown in the video and provide numerical data about the objects' positions over time.

Two videos were analyzed: a thrown hammer, and a falling toy parachute. Each video was taken such that the motion happens squarely within the plane perpendicular to the camera's central viewing direction. In the hammer video, the center of mass of the hammer is known and identified, and the hammer spins as it traces a visible parabolic arc through the air. In the parachute video, the toy is dropped from a height while the chute is not open. As the toy falls, the parachute begins to open, until it maintains a virtually constant state of openness during the rest of the fall. In both videos, a meter stick is in shot, approximately in the same plane as the object motion.

Figure 1: Diagram of hammer's motion

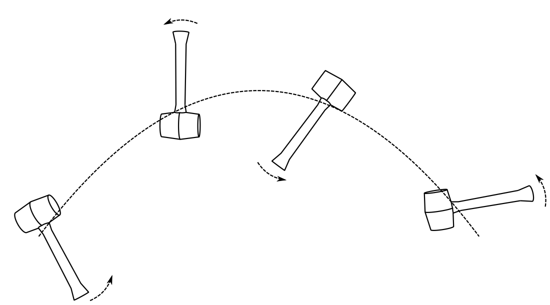
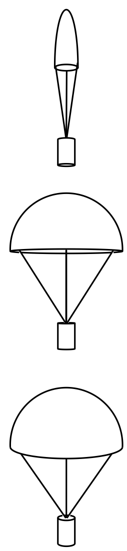


Figure 2: Diagram of toy's motion



Information about the motion of two pendulums was also provided. A table of the angle at different times was given for a pendulum of small angles and a pendulum of relatively larger angles. As this data is already in numerical form, it need not have been processed in the way that the video data was.

3 Data Extraction

The Logger Pro software is capable of performing motion tracking, but the open source motion tracking software Blender 3D was used in this case. For the hammer, the center of mass was known to be the center point at which the head and handle meet; this point was tracked throughout the video, as well as another point at the end of the handle. For the parachute, the bottom of the toy was tracked during its fall. The data outputted was a list coordinates in pixels of the tracked points at each given frame.

A meter stick was present in each video; the number of pixel lengths between each end of the stick was calculated and used to convert the pixel coordinates into units of meters. The frame rate of each video was known to be 29.97 **Hz** , which was used as a conversion factor from frame number to length of time elapsed in seconds.

From these procedures, a dataset was obtained that was able to list metric locations of each object at different instances in time.

Figure 3: Hammer center of mass position

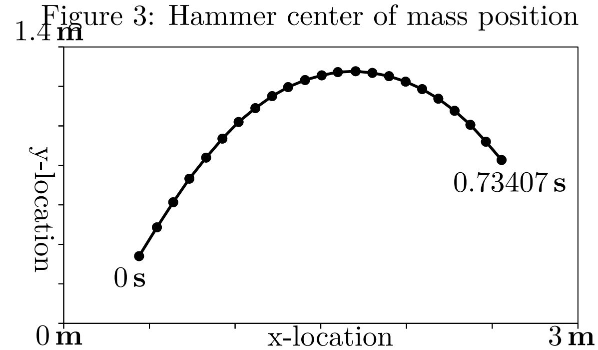


Figure 4: Hammer handle end position

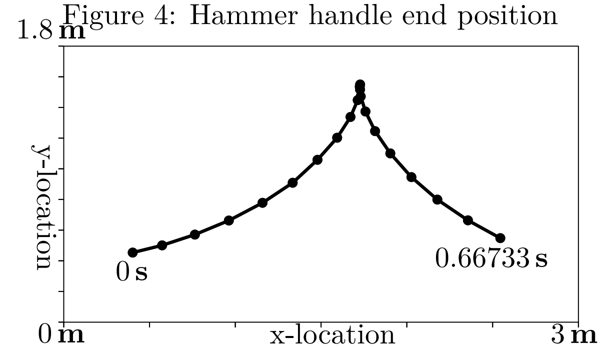
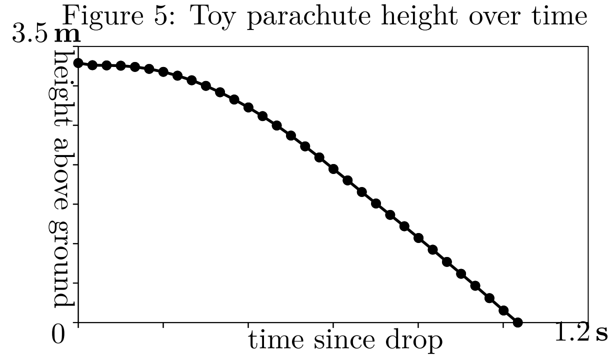


Figure 5: Toy parachute height over time



The horizontal position of the toy parachute over time was recorded, but the change was very insignificant. The motion of the toy was almost entirely vertical.

4 Data Analysis

4.1 Hammer

Projectile motion in a uniform gravitational field and no air resistance is known to result in a parabolic arc, where horizontal movement is of constant velocity and vertical movement is under constant acceleration. While the hammer was thrown on Earth in a spherical and slightly uneven gravitational field with air resistance present, the movement domain is small enough to approximate the gravity as uniform, and the object was small and heavy enough to make air resistance somewhat negligible.

A quadratic regression over the vertical location of the hammer's center of mass with respect to time was performed with the form of $y = \frac{-1}{2}gt^2 + v_0t + y_0$. The following coefficients were created with a 96.875% confidence interval.

Coeff.	Value	Error
g	9.9990470 $\mathbf{m}/\mathbf{s}^2$	$\pm$0.0799659 $\mathbf{m}/\mathbf{s}^2$
v_0	4.6490049 $\mathbf{m}/\mathbf{s}$	$\pm$0.0371797 $\mathbf{m}/\mathbf{s}$
y_0	0.1996834 $\mathbf{m}$	$\pm$0.0099750 $\mathbf{m}$

The standard error of this regression is 0.0008784 $\mathbf{m}$.

The error of all coefficients here is determined only based on the uncertainty of the regression; the error in the measurements is not accounted for. One possible reason for the value of g differing from the commonly known 9.81 $\mathbf{m}/\mathbf{s}^2$ is that the meter stick was some distance behind the hammer, making it seem like the hammer traveled farther in less time and also accelerated faster.

The same process was done with the horizontal component of movement of the hammer's center of mass. Because our model neglects horizontal acceleration, a linear regression was used. Again, confidence intervals used in this paper are with a confidence level of 96.875% (equal to $1 - 1/32$).

Coeff.	Value	Error
v_0	2.8913150 $\mathbf{m}/\mathbf{s}$	$\pm$0.0104441 $\mathbf{m}/\mathbf{s}$
x_0	0.3481827 $\mathbf{m}$	$\pm$0.0047782 $\mathbf{m}$

The standard error of this regression is 0.0000215 $\mathbf{m}$.

To test if these regressions are good models of the movement, the residual plots for each are shown.

Figure 6: Vertical position regression residual

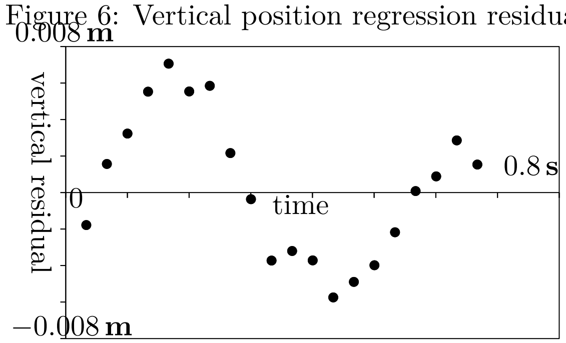
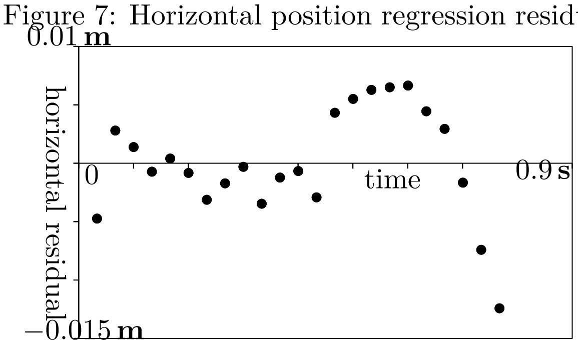


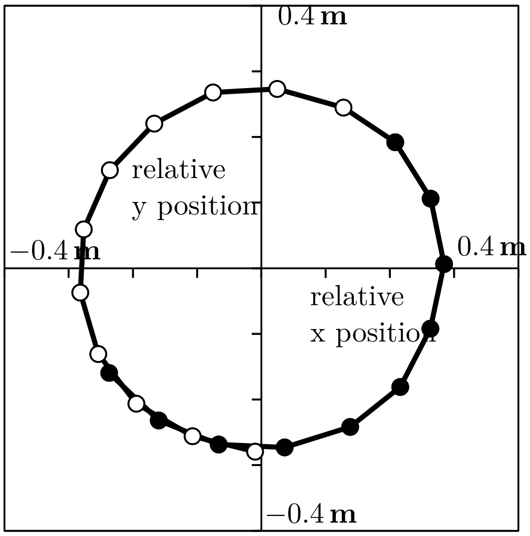
Figure 7: Horizontal position regression residual



The residual plots seem to trace out patterns, indicating that the quadratic and linear regressions used do not tell the full story. The horizontal residual decreasing towards the end could be a result of air resistance making the traveled distance progressively less as the resistive force has more time to act. It is also possible that the true position of the center of mass of the hammer was actually slightly off from where we tracked, resulting in a spinning acceleration in the data.

To gain extra insight, the endpoint of the hammer's handle was tracked. The motion of the handle from a global frame of reference is complicated, so shown is the graph of the position of the handle relative to the tracked center.

Figure 8: Handle relative to center



Since the graph overlaps itself, the points that happened in the latter half of the trajectory are unfilled for clarity.

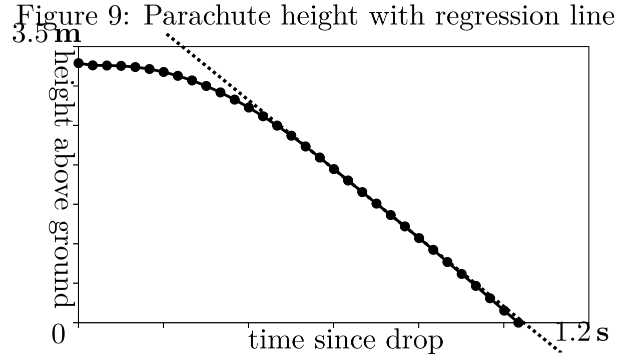
The points on this graph are approximately evenly spaced, suggesting that, relative to the center of mass, the end of the handle undergoes uniform circular motion.

4.2 Toy Parachute

The hammer from the previous section was heavy and small enough to make air resistance's effect on it very small. The toy parachute, however, is light and has a large area, making air resistance's effects on it very noticeable.

Objects undergoing air resistance and an other constant force tend to accelerate at a decreasing rate, approaching a constant velocity over time. This is apparent in the data by applying a linear regression over the latter half of the data.

Figure 9: Parachute height with regression line



The line was of equation $y = -v_0 t + y_0$. Here are the values of the coefficients:

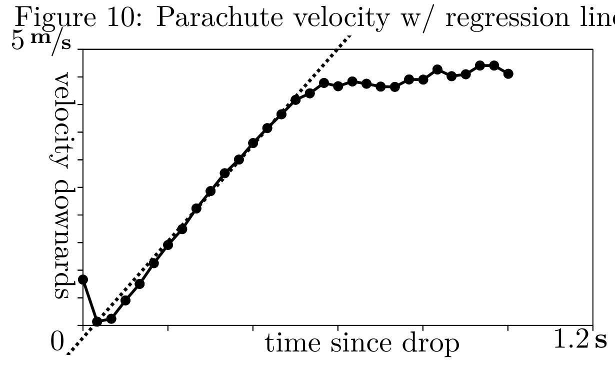
Coeff.	Value	Error
v_0	$4.4458112\,\mathbf{m/s}$	$\pm 0.0404143\,\mathbf{m/s}$
y_0	$4.6206172\,\mathbf{m}$	$\pm 0.0315577\,\mathbf{m}$

The value of v_0 here represents the constant velocity that the toy moves under air resistance, the terminal velocity.

The y_0 value here does not represent the initial height of the toy; it instead represents the height that it would've been if the entire fall was of constant velocity, respecting the second half of the data and neglecting the first half.

However, because the motion starts slowly and the parachute needs time to open, the motion begins with roughly constant acceleration, apparent in the data by applying a linear regression over the *velocity* over time with the first half of the data.

Figure 10: Parachute velocity w/ regression line



The first point was caught when the parachute was still being dropped, and was neglected in the regression.

The line was of equation $v = gt + v_0$. Here are the values of the coefficients:

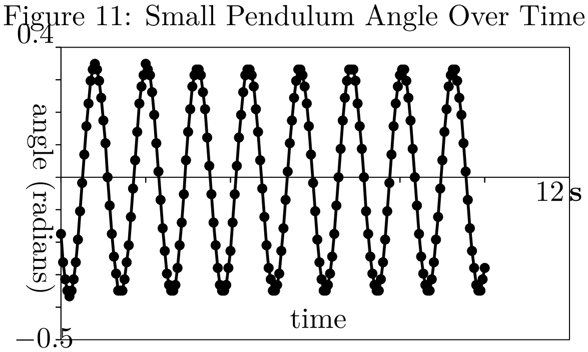
Coeff.	Value	Error
g	$9.2923158\,\mathrm{m/s^2}$	$\pm 0.4022483\,\mathrm{m/s^2}$
v_0	$-0.4082234\,\mathrm{m/s}$	$\pm 0.1065314\,\mathrm{m/s}$

The value of g is somewhat close to the value of gravitational acceleration, but is still off considering the error range not including $9.81\,\mathrm{m/s^2}$. The value of g predicted is lower than it should be, which is possibly a result of the drag being slightly present even in the selected range.

4.3 Pendulum of Small Amplitude

The motion of the pendulum was provided in the form of a table with the angle at different times. Here is a graph:

Figure 11: Small Pendulum Angle Over Time



If this pendulum is modeled as a frictionless oscillator, it is described by a sinusoidal equation.

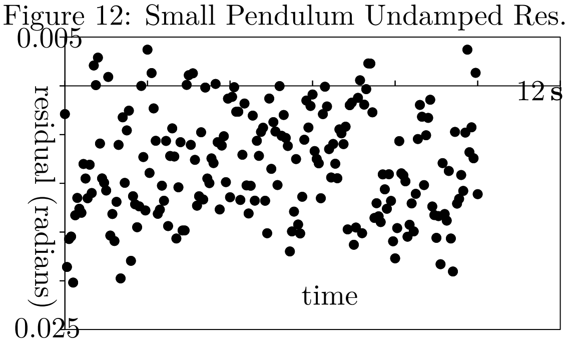
$$\theta(t) = A\sin(\omega t + \phi)$$

A least squared regression solver was used to find values that satisfy this equation. Here are the results:

Value	Symbol	Value
Amplitude	A	-0.3463644
Angular Frequency	ω	$5.2003203\,\mathbf{Hz}$
Offset	ϕ	0.5184781

Due to limitations with using a numerical solver, the error of each value cannot be found, but the total standard error of the approximation is 0.0006966 radians. Here is a residual graph:

Figure 12: Small Pendulum Undamped Res.



While the residual plot is chaotic, it is shifted down. This implies that the pendulum in fact oscillates off center from what is considered $\theta = 0$. The rest was performed again using a shifted equation.

$$\theta(t) = A\sin(\omega t + \phi) + s$$

Here are the new numbers:

Value	Symbol	Value
Amplitude	A	-0.3457983
Angular Frequency	ω	$5.1998443\,\mathbf{Hz}$
Offset	ϕ	0.5207339
Shift	s	-0.0081575

The standard error of this approximation is 0.0003841 radians, better than the previous model that didn't incorporate shift. The residual plot is almost the same, but instead centered around 0.

The sinusoidal model is good in this case, but in reality, pendulums decay in amplitude over time. An equation can reflect this decay:

$$\theta(t) = A\exp(-\alpha t)\sin(\omega t + \phi) + s$$

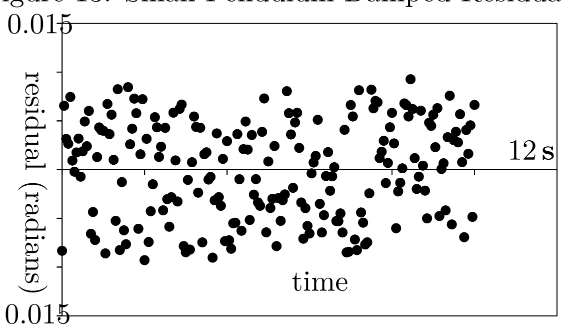
Using the solver, the following values were returned:

Value	Symbol	Value
Amplitude	A	-0.3510799
Angular Frequency	ω	$5.1997997\,\mathbf{Hz}$
Offset	ϕ	0.5206716
Shift	s	-0.0082146
Dampening	α	$0.0030033\,\mathbf{Hz}$

The standard error is 0.00035179 radians, only marginally smaller than the 0.0003841 seen before. Notably, the amplitude in the decaying model is higher than the amplitude in the undamped model; this makes sense, because the undamped model must be set lower to compensate for the overall lower amplitude, while the damped model can have a high amplitude value, while the exponential factor will handle the decreasing nature of the observed amplitude.

Here is the residual plot for the damped model:

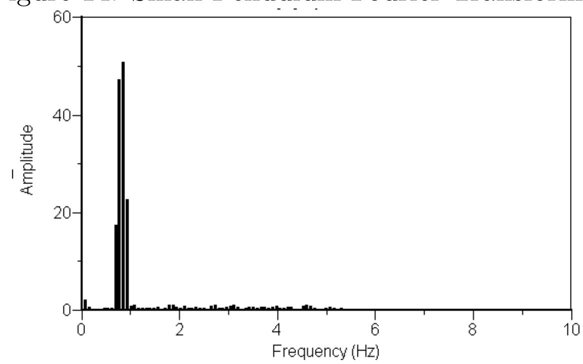
Figure 13: Small Pendulum Damped Residuals



The residual plot is sufficiently chaotic, suggesting that the damped model is a good depiction of the data.

An additional method of demonstrating the movement's likeness to sinusoidal motion is to perform a Fourier Transform. Logger Pro was used to perform this transform to produce this graph:

Figure 14: Small Pendulum Fourier Transform

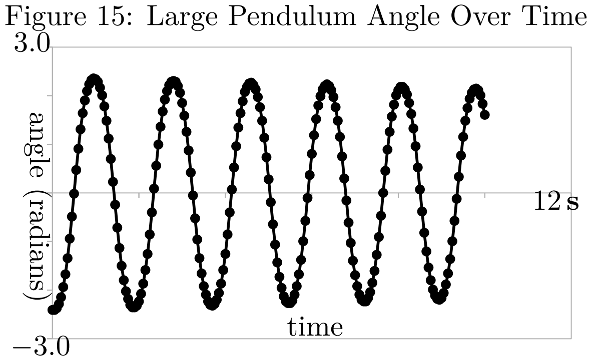


The Fourier Transform, in a loose sense, changes the graph of angle over time into a graph of amplitude over frequency, illustrating the extent to which each frequency makes up the total movement. The fact that most of the area of the graph is located at one point is reassuring, implying that the motion is pretty much consistent of one single frequency, making it a near perfect sine wave.

4.4 Pendulum of Large Amplitude

The same procedure as above was done for a pendulum of relatively larger amplitude. Here is the graph of its motion:

Figure 15: Large Pendulum Angle Over Time

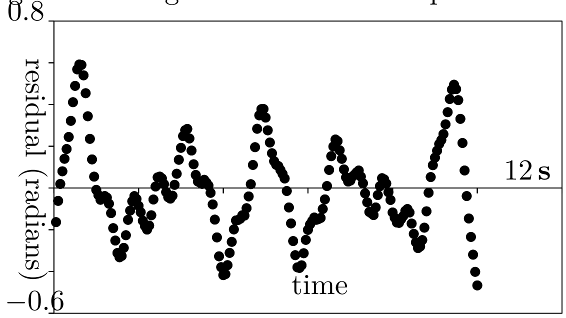


Using the undamped model $\theta(t) = A\sin(\omega t + \phi) + s$, here are the values:

Value	Symbol	Value
Amplitude	A	-2.3332762
Angular Frequency	ω	$-3.5477394\,\mathbf{Hz}$
Offset	ϕ	1.9993896
Shift	s	0.0150121

The standard error is 0.01561118 radians, which is significantly worse than for the small pendulum. The residual plot gives insight into why.

Figure 16: Large Pendulum Undamped Residual



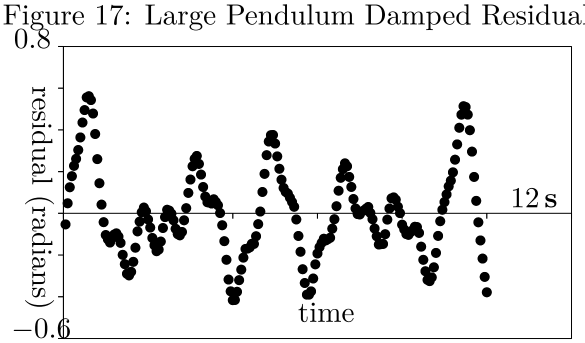
The residuals trace out a clear path, indicating that the motion is in fact not very sinusoidal in nature. The process was repeated even with damped oscillation in mind:

$$\theta(t) = A\exp(-\alpha t)\sin(\omega t + \phi) + s$$

Value	Symbol	Value
Amplitude	A	−2.4482355
Angular Frequency	ω	−3.5465201 **Hz**
Offset	ϕ	1.9929790
Shift	s	0.0096747
Dampening	α	0.0185819

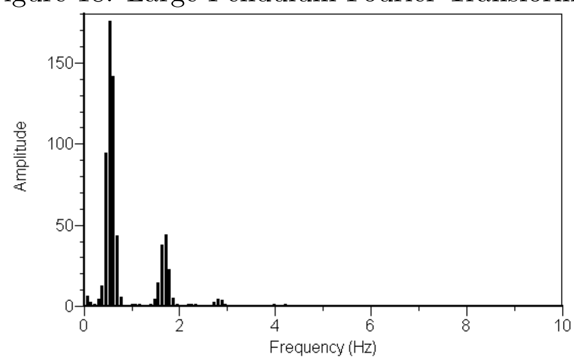
The standard error is still not great at 0.0152454 radians. In the residual plot, a very similar pattern is seen.

Figure 17: Large Pendulum Damped Residual



The best conclusion from this analysis is that the motion relatively not close to being a sine wave. For further confirmation of this, another Fourier Transform was performed with Logger Pro on this data, similar to as was done with the small pendulum.

Figure 18: Large Pendulum Fourier Transform



More than one peak is apparent, and these peaks are not as narrow as the peaks of the smaller pendulum's graph. This suggests that the motion is not sinusoidal, and in fact can be approximated better with two sine waves, but not perfectly without a different type of model entirely.

5 Conclusion

After analyzing the data here, most of it was somewhat well described by simple Newtonian models. Projectile motion was approximated with constant acceleration, and turned out to yield a good representation of the motion. The parachute with great air resistance demonstrated both quadratic and linear motion during different time intervals. The small amplitude pendulum demonstrated sinusoidal motion quite well. However, the pendulum of large amplitude did not fall cleanly into a sinusoidal approximation. The models used here largely did well, except not in all circumstances.

The data was analyzed from different angles; the pendulums were analyzed with sinusoidal regression and Fourier Transforms, yielding two complementary perspectives that each gave insight into the nature of the motion.