Oscillations

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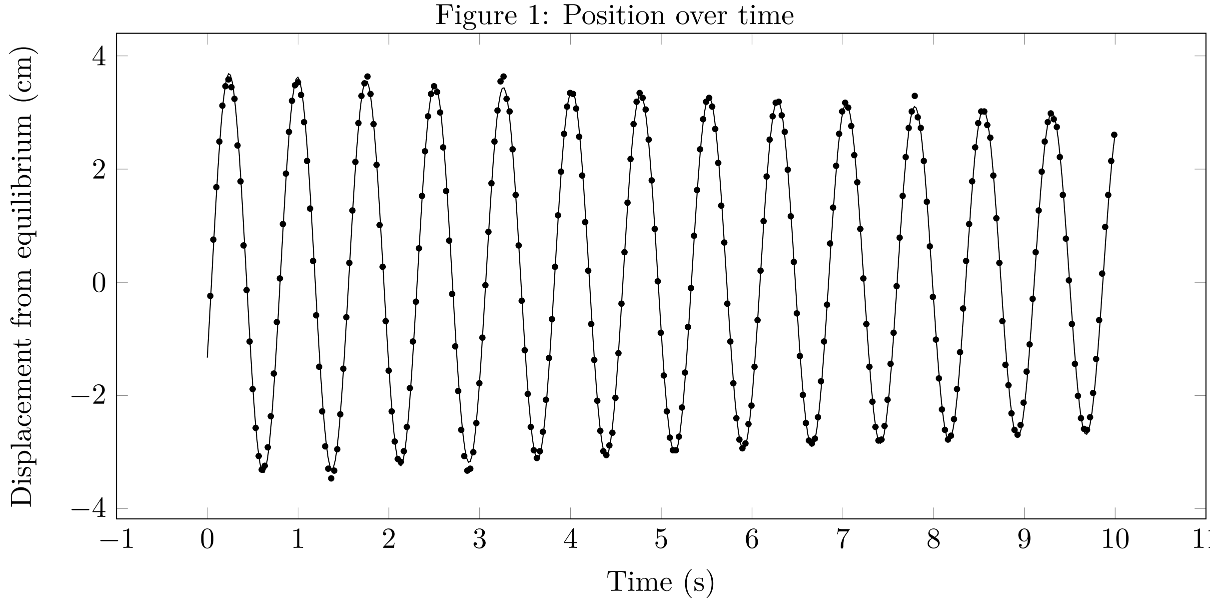
Different types of oscillating motion are to be observed and analyzed in this lab. An approximately Hookean spring will be compared to a non-Hookean rubber band in oscillation.

1 Springs

A spring was hung from a fixed point and on it was hung a mass of 0.231 **kg**. The mass and spring end was pulled down by a couple centimeters and released while the position of the mass and force at the top of the spring were measured over time.

Here is a graph of the displacement over time with a dampened harmonic motion curve superimposed. The origin of displacement in this case is the equilibrium position of the mass while it rests upon the spring.

Figure 1: Position over time

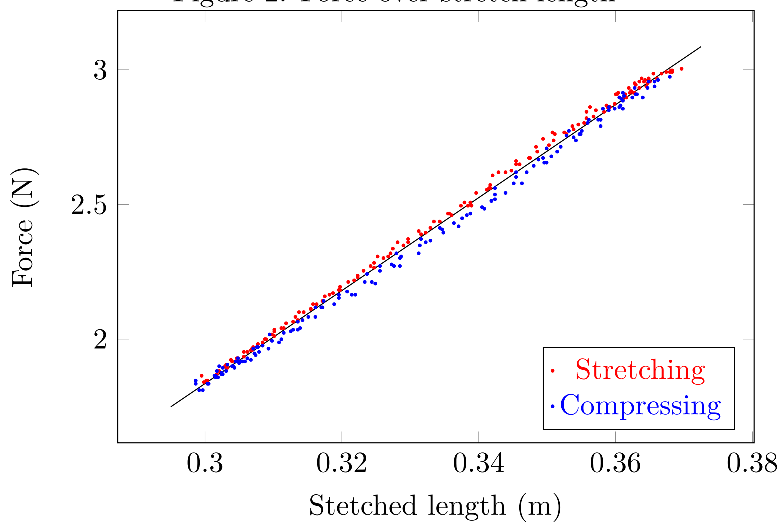


The oscillating motion stays relatively stable in amplitude, but a slow decrease over time is observed due to resistive forces.

The angular frequency of the motion is estimated at 8.319 **Hz**. Since this is a non-linear model, it is difficult to obtain the error of this estimate.

Here is a graph of the force as a function of the length of the stretched spring with a linear regression superimposed. The length here refers to the total length of the spring at any given time.

Figure 2: Force over stretch length



The data is evidently quite linear. The slope of this graph can be used to predict the spring constant to be $(17.257 \pm 0.1275)\,\mathrm{N/m}$ (with 96.875% confidence).

It is interesting to note that the force is higher while stretching than for compressing parts of the motion. This suggests that the work done by gravity onto the spring while stretching is more than the work that is returned from the spring. This is in line with the dampening of the motion, suggesting that energy is being taken through friction and other resistive forces.

For a perfectly Hookean system, the spring constant would equal the mass of the load on the spring times the square of the angular frequency. Based on the spring constant calculation and the angular frequency estimate of $8.319\,\mathbf{Hz}$, the mass of the system would independently be calculated to be about $0.249\,\mathbf{kg}$. The error of this mass estimate due to the error in the spring constant alone is $0.001842\,\mathbf{kg}$.

While the load on the end of the spring was in fact $0.231\,\mathbf{kg}$, the spring itself has a mass of $0.015\,\mathbf{kg}$. Skipping the relevant calculus, a good approximation equates the motion of the spring and mass system to the motion of a massless spring of the same modulus and a mass with additionally one third the mass of the spring.

Using this approximation, $0.005\,\mathbf{kg}$ of the mass can be removed from the calculated mass to yield an adjusted calculated mass of $0.244\,\mathbf{kg}$. The error in the mass calculation, due to the spring constant error alone, of $0.001842\,\mathbf{kg}$ is not enough to encompass the true value of $0.231\,\mathbf{kg}$. This suggests error of other types, possibly in the angular frequency measurement or other sources.

If we were to instead use the true effective mass of $0.231\,\mathbf{kg} + 0.005\,\mathbf{kg}$ to calculate the spring constant independently of the graph used to calculate it above, we would get an estimate of $16.3325\,\mathrm{N/m}$. This leads to two ways of estimating the spring constant, one using force over displacement readings and one using the angular frequency and known mass.

This spring and mass oscillation experiment was repeated with different masses. For each trial, the spring had an unstretched length of 0.20 **m** and mass of 0.015 **kg**. Here is a table of data describing each case:

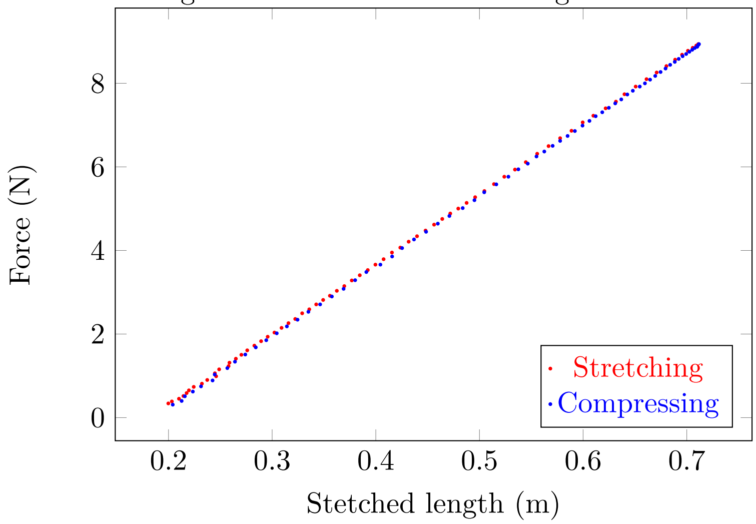
Table 1: Various trials and data

Mass on the end (kg)	Equilibrium length (m)	Angular frequency (Hz)	Spring modulus based on frequency (N/m)	Spring modulus based on force (N/m)
0.131	0.28	10.88	16.1	16.9
0.172	0.30	9.59	16.3	17.0
0.231	0.335	8.32	16.3	17.3
0.311	0.382	7.19	16.3	16.9
0.431	0.45	6.15	16.5	16.9
0.731	0.622	4.79	16.9	17.3
0.931	0.735	4.28	17.1	17.5

If the spring was perfectly Hookean, the spring modulus would be constant and perfectly independent from the equilibrium length. The fact that the spring modulus seems to increase with longer stretch lengths suggests that the spring is not perfectly Hookean. However, the amount by which the modulus changes is relatively small, suggesting that Hooke's law is a decent approximation for the system.

This can be further confirmed by measuring the force exerted at a variety of different stretch lengths. Here is a graph of force over stretched length of the spring, with a much larger domain than in Figure 2.

Figure 3: Force over stretch length



The linear nature of this graph suggests that Hooke's Law is a good approximation for the spring. The slope of this line is 16.881 **N/m**, reasonably in line with the various calculated spring moduli in the table above.

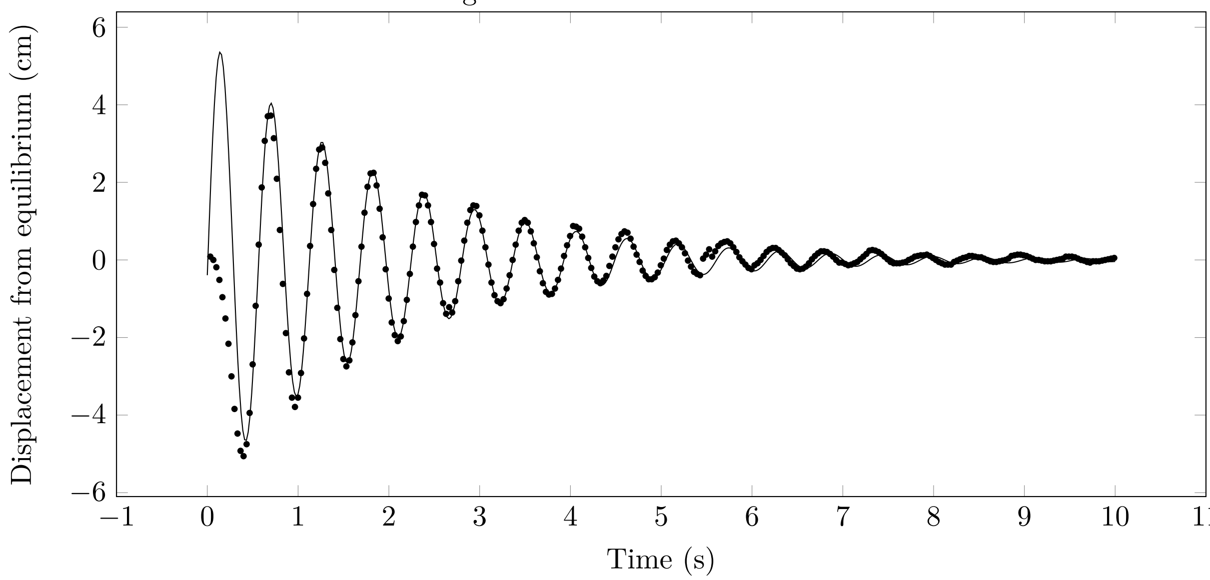
It is again interesting to note that upon close examination, the stretching forces tend to be slightly greater than the compressing forces, pointing to the loss of energy over oscillation.

2 Rubber bands

The same process as above was performed, except with a rubber band in place of a spring. The rubber band was, with one end fixed, stretched by a mass of 0.231 **kg** hung onto it.

Here is a graph of displacement over time with a dampened harmonic motion curve superimposed. The origin of displacement is again set to the equilibrium position of the mass while it rests upon the rubber band.

Figure 4: Position over time



It should be noted that the beginning of the curve likely does not fit due to how the oscillating motion was initiated, so the section of data before the first local minimum was not considered in the curve fit.

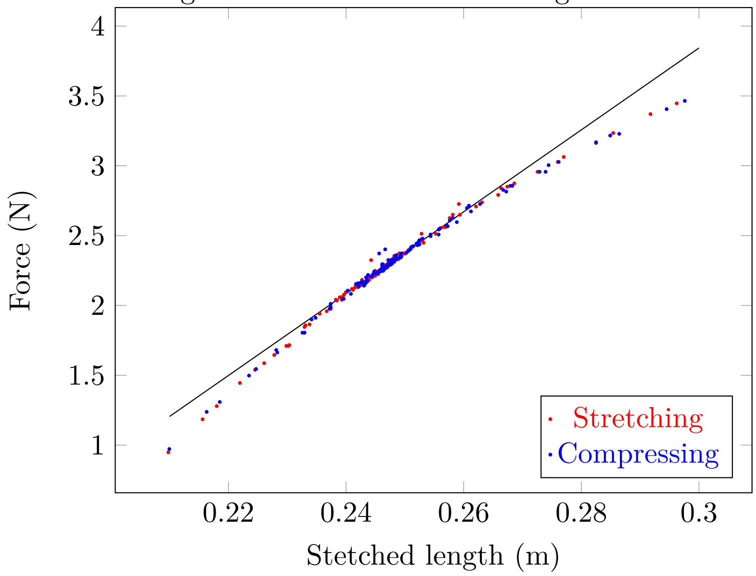
The motion is quickly dampened over time. In this case, the exponential decay constant was 0.504 **Hz**, making the oscillation amplitude have a half life of only 1.373 **s**.

The angular frequency is estimated at 11.216 **Hz**. Again, since this is a non-linear model, the statistical error is difficult to determine.

Given the mass on the rubber band was 0.231 **kg** and the mass of the band itself was 0.002 **kg**, it can be estimated that the elastic modulus of the rubber band around this oscillation was 29.311 **N/m**.

The force over stretch length can be plotted again here. This again refers to the total length of the rubber band at any given time.

Figure 5: Force over stretch length



The superimposed line is not a linear regression of this data; it is a line of slope 29.311 N/m at an arbitrary intercept, placed to highlight the non-linear nature of this graph. The slope, which is the elastic modulus estimate from the oscillating data, does seem to roughly align with what a linear regression would look like on this non-linear data.

The non-linear nature of the curve suggests that a rubber band does not have a Hookean restoring force. This suggests that the initial assumption of the dampened harmonic curve fit being applicable to this data was in fact a false assumption.

Further evidence can be found by comparing the elastic modulus from oscillations around different stretch lengths based on different forces applied.

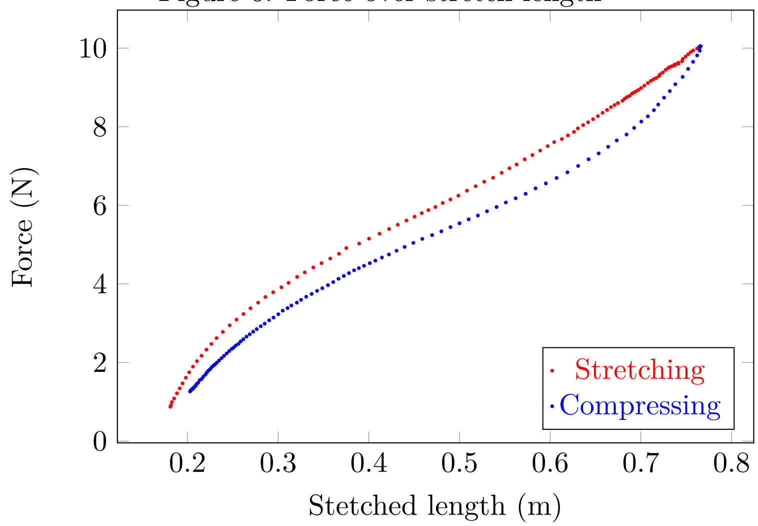
Table 2: Various trials and data

Mass on the end (kg)	Equilibrium length (m)	Angular frequency (Hz)	Elastic modulus based on frequency (N/m)	Elastic modulus based on force (N/m)
0.171	0.225	14.4	35.46	36.9
0.231	0.247	11.2	28.98	29.6
0.331	0.305	7.56	18.92	19.4
0.431	0.362	5.91	15.05	15.4
0.531	0.450	5.08	13.70	13.6
0.631	0.557	4.69	13.88	13.6
0.731	0.635	4.92	17.69	16.5

Based on these oscillating tests, the elastic modulus does depend significantly on the stretch length.

Taking it a step further, the force can be graphed over the stretch length.

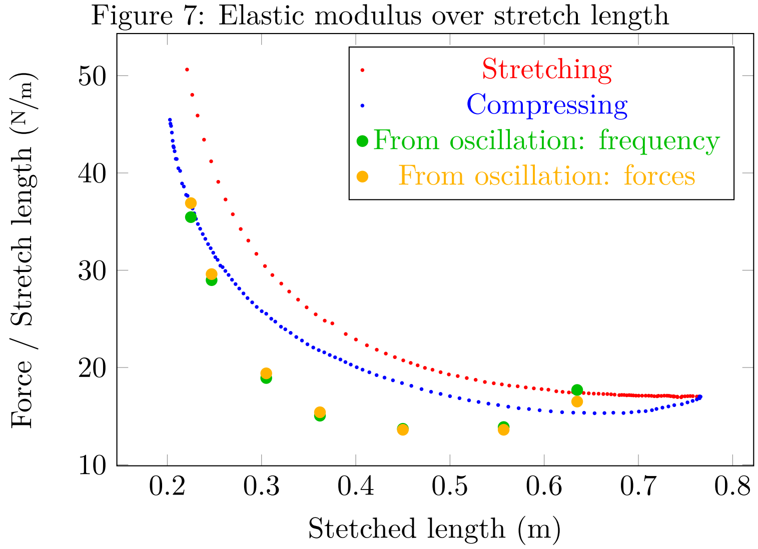
Figure 6: Force over stretch length



As can be seen, the force over length curve is not linear at all. Furthermore, the large difference between stretching and compressing forces agrees with the very quick dampening of the oscillation, because more work by gravity is done while stretching than is returned from the spring while compressing.

This data can be compared to the effective spring constants found by various oscillating motion setups. Here is a graph of the force divided by the displacement from the equilibrium length, over stretched length. The calculated elastic moduli based on the oscillating motion is also plotted, including one point for each calculation method.

Figure 7: Elastic modulus over stretch length



While the data looks close numerically at the least and most stretched states, there is a discrepancy for medium lengths. This could suggest that the rubber band in oscillation actually behaves differently from the rubber band when being pulled over the course of about 7 seconds, which is the time interval over which the force / distance data was collected.