Rice University
PHYS 112

RC Circuits

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March 29, 2021

An electronic circuit will be analyzed. In the circuit, a capacitor and resistor are connected to a battery to create a time-dependent voltage over the capacitor.

1 Circuit

1.1 Construction

A circuit was constructed with the ability to both charge and discharge a capacitor. The current was regulated by a resistor to ensure the capacitor charged and discharged over a significant interval of time.

The instructor provided measurements and recordings of such a circuit. However, I thought it'd be fun to build the circuit and do the measurements myself.

Below is a schematic of the circuit I built:

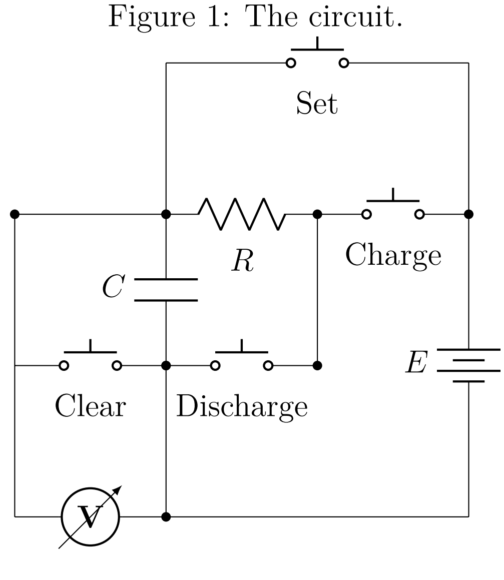
When disconnected, voltage E was measured to be $(3.306 \pm 0.027)\,\mathbf{V}$.

There are four switches in this circuit:

- Charge — Allows the capacitor to be charged through the resistor.
- Discharge — Allows the capacitor to be discharged through the resistor.
- Set — Immediately charges the capacitor to voltage E.
- Clear — Immediately discharges the capacitor.

The circuit was put together on a solderless breadboard with hobby-grade components. The voltmeter used was an Arduino Uno with analog signal input.

Figure 1: The circuit.



1.2 Expected behavior

When the Discharge button is closed, the capacitor's ends will effectively be joined by the resistor. The current I will be the same across the capacitor and resistor. The voltage V across the capacitor will be negative of that over the resistor.

- Ohm's law will apply on the resistor: $-V = IR$
- The relation for a capacitor will also apply: $I = C\frac{dV}{dt}$

Solving for I and setting both expressions for I equal to each other,

$$V = -RC\frac{dV}{dt}$$

Thus, with the capacitor charged initially at voltage V_0,

$$V(t) = V_0 \exp\left(-\frac{t}{RC}\right) \tag{1}$$

When the Charge button is closed instead, the capacitor is effectively entered into a circuit with the resistor and battery in series. The differential nature is the same although negative, since the end voltage is higher than the initial, with equilibrium voltage E.

$$V(t) = E\left(1 - \exp\left(-\frac{t}{RC}\right)\right) \tag{2}$$

The capacitor, when linked with the resistor in series, can act as an electronic time-keeper since its voltage changes predictably over time. Given the resistance, capacitance, starting and equilibrium voltage, the voltage over the capacitor can be deduced for any given moment in time. Furthermore, if a value like resistance or capacitance is otherwise unknown, the voltage over the capacitor over time can be used to fill in the information gaps. Both of these facts were explored and put to the test in this lab.

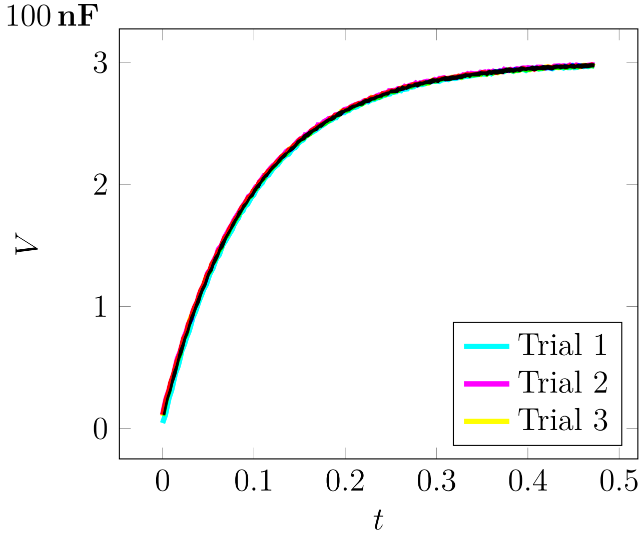
2 Experimentation

The circuit was used in several instances with different values of resistance R and different capacitor configurations. First, a $1\,\mathbf{M\Omega}$ resistor was used with a $100\,\mathbf{nF}$ capacitor. With a multimeter, the values were checked to be $1.004\,\mathbf{M\Omega} \pm 0.01506\,\mathbf{M\Omega}$ and $96.4\,\mathbf{nF} \pm 7.712\,\mathbf{nF}$.

The capacitor was charged and discharged each a total of 3 times.

Here is the graph of the capacitors voltage over time whilst charging:

Figure 2: Charging, $V(t)$; $R = 1\,\mathbf{M\Omega}$, $C = 100\,\mathbf{nF}$



A regression of the following equation was performed on all three datasets:

$$\hat{V}(t) = (\hat{E} - \hat{V}_0)\left(1 - \exp\frac{-t}{\hat{RC}}\right) + \hat{V}_0$$

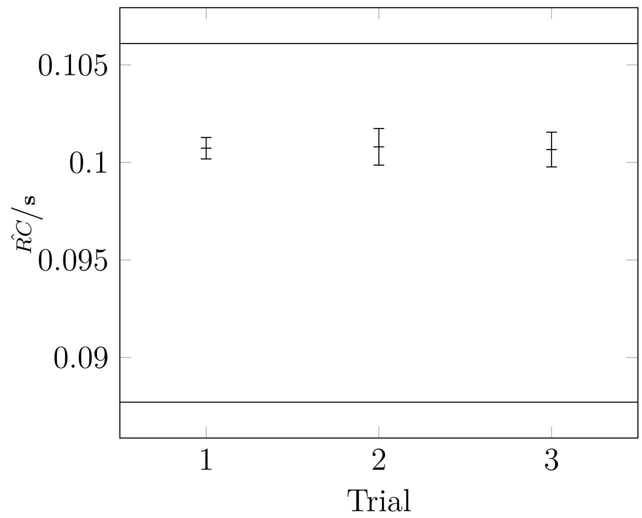
This regression equation is similar to equation 2, but it has an extra corrective factor $\hat{V}_0$ to account for the fact that in these experiments, the starting voltage over the capacitor was not truly 0.

To create a rough confidence interval for the value of $\hat{RC}$, the data was shifted by the maximum voltage measured in order to place the asymptote at 0, and a linear regression was performed, with a confidence interval over the slope. Each confidence interval that follows is at 96.875%, but this is merely the confidence interval over the linearized data, so this percentage does not reflect a true reading of confidence.

Here are the resulting values of $\hat{RC}$ from charging:

Table 1: Charging, $\hat{RC}$; $R = 1\,\mathbf{M\Omega}$, $C = 100\,\mathbf{nF}$

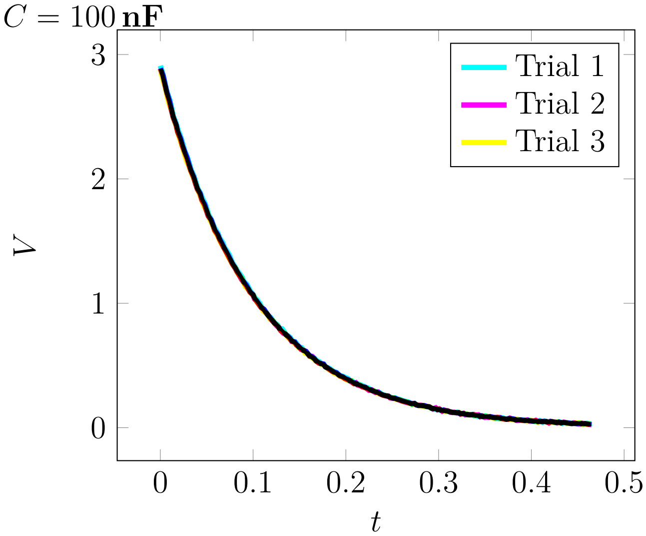
Trial	$\hat{RC}/\mathrm{s}$	Error/s
1	0.10073	±0.00055
2	0.10080	±0.00094
3	0.10066	±0.00089



In the above figure, the horizontal lines at the top and bottom are placed at the values of RC calculated from the multimeter readings of the resistor and capacitor.

The same procedure was performed whilst the capacitor was discharged. Here is the graph:

Figure 3: Discharging, $V(t)$; $R = 1\,\mathbf{M\Omega}$, $C = 100\,\mathbf{nF}$



For discharging, the regression equation was as follows:

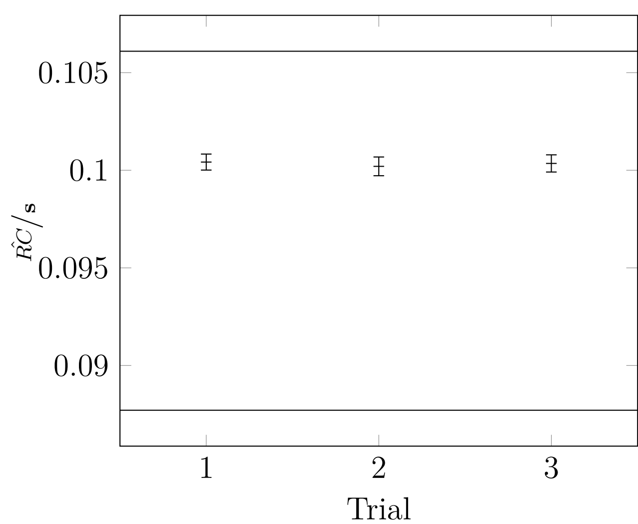
$$\hat{V}(t) = \hat{V}_0 \exp \frac{-t}{\hat{RC}}$$

This equation is true to equation 1.

The process of calculating was the same as before, except the data did not need to be shifted by a corrective factor for linearization, since the limit is known to be 0. Here are the values of $\hat{RC}$ from discharging:

Table 2: Discharging, $\hat{RC}$; $R = 1\,\mathbf{M\Omega}$, $C = 100\,\mathbf{nF}$

Trial	$\hat{RC}/\mathbf{s}$	Error/$\mathbf{s}$
1	0.10042	0.00041
2	0.10020	0.00048
3	0.10035	0.00044



For both charging and discharging, the values of $\hat{RC}$ agree as they all fall in each others' error margins.

2.1 Varying Resistance

The experiment was repeated 5 more times, resulting in a total of 36 trials. With each new experiment, a new resistor or set of resistors was used.

Table 3: Resistors used

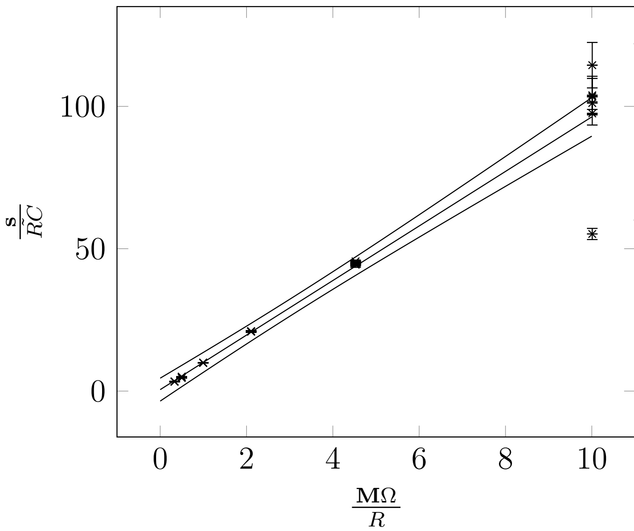
Experiment	$R/\mathbf{M\Omega}$	Error/$\mathbf{M\Omega}$
1	1.004	0.0150
2	2.005	0.0301
3	3.007	0.0451
4	0.475	0.0072
5	0.2209	0.0033
6	0.0999	0.0015

It is expected that $\hat{RC}$ is about proportional to R by factor C. It is useful to look at the relationship of the inverses, $\frac{1}{\hat{RC}}$ vs $\frac{1}{R}$. This is because it makes it easy to account for a non-ideal voltmeter, which acts similarly to an ideal voltmeter with a resistor in parallel. Skipping the derivation, it is seen ideally, that for some measured $\tilde{RC}$, with a voltmeter with parallel resistor of resistance R_m,

$$\frac{1}{\tilde{RC}} = \frac{1}{RC} + \frac{1}{R_m C}$$

This is effectively an equation for $\frac{1}{\tilde{RC}}$ in terms of a linear function of $\frac{1}{R}$.

With known $\frac{1}{\hat{RC}}$ and $\frac{1}{R}$ for each of the 36 trials, the following graph was produced.



The center line is a linear regression, whilst the outer two lines enclose a Working-Hotelling confidence band of 96.875%.

The intercept of the linear regression is at 0.5084 **s** . Dividing by the capacitance of 96.4 **nF** , the regression alone indicates that $R_m = 5.274\,\mathbf{M\Omega}$. This is fairly reasonable; the Arduino is likely not able to measure voltage without taking in a little bit of current, and 5.274 **MΩ** is a high resistance value that indicates the smallness of this current allowed.

However, the error in the linear regression indicated by the confidence band is enough to completely encompass the origin. Thus, while 5.274 **MΩ** seems plausible, it is not possible from this data alone to conclude even that $R_m \neq 0$.

2.2 Measuring Capacitance

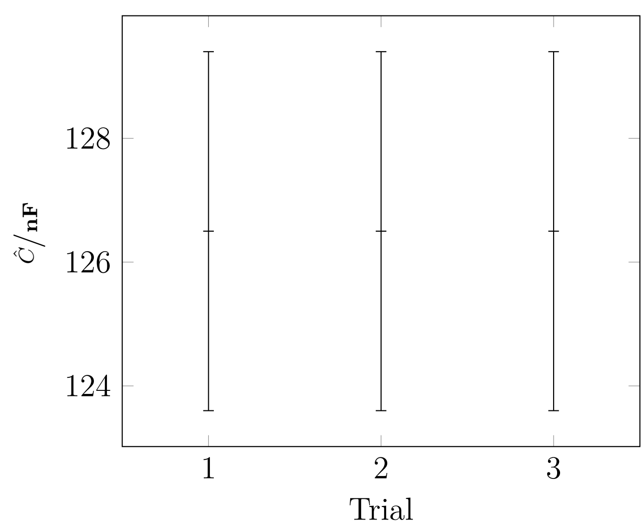
If resistance R and the exponential rate of voltage RC are known, the capacitance can be calculated by division.

The 100 **nF** capacitor was placed in parallel with a 47 **nF** capacitor. The total capacitance was measured to be 147.2 **nF** ± 11.776 **nF** .

Here are the resulting of $\hat{RC}$ divided by R for charging:

Table 4: Charging, $\hat{C}$; $R = 1\,\mathbf{M\Omega}$, $C = 147\,\mathbf{nF}$

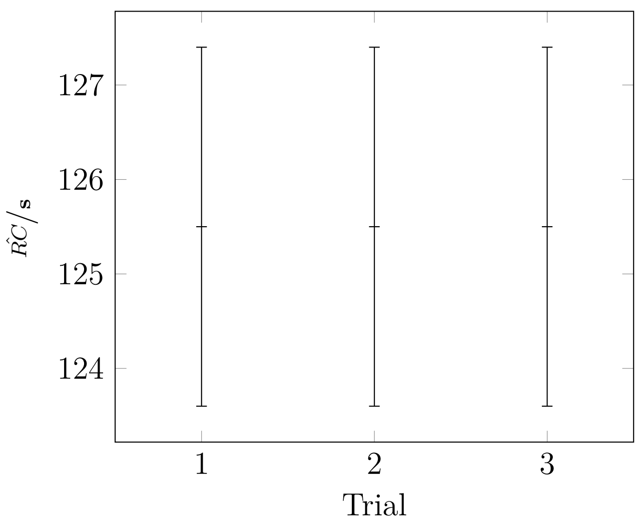
Trial	$\hat{C}/\mathbf{nF}$	Error/**nF**
1	126.5	±2.9
2	126.5	±2.9
3	126.5	±2.9



And here are the results for discharging:

Table 5: Discharging, $\hat{C}$; $R = 1\,\mathbf{M\Omega}$, $C = 147\,\mathbf{nF}$

Trial	$\hat{RC}/\mathbf{s}$	$\text{Error}/\mathbf{s}$
1	125.5	±1.9
2	125.5	±1.9
3	125.5	±1.9



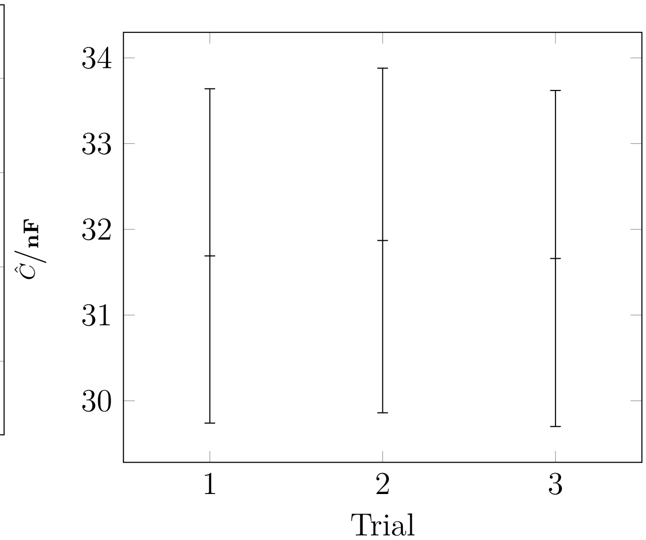
The results consistently gave a capacitance value of around $126\,\mathbf{nF}$, contradicting the direct measurement of around $147\,\mathbf{nF}$ and at least around $135\,\mathbf{nF}$. This is likely due to something having gone awry in the circuitry for this run, like a misplaced wire or incomplete connection.

The capacitors were reorganized to be in series. The capacitance was measured to be $31.18\,\mathbf{nF} \pm 2.4944\,\mathbf{nF}$.

Here are the results for charging:

Table 6: Charging, $\hat{C}$; $R = 1\,\mathbf{M\Omega}$, $C = 31\,\mathbf{nF}$

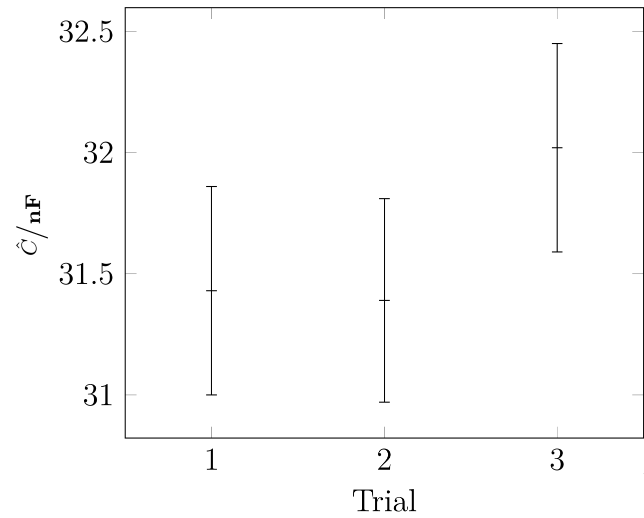
Trial	$\hat{C}/\mathbf{nF}$	$\text{Error}/\mathbf{nF}$
1	31.69	±1.95
2	31.87	±2.01
3	31.66	±1.96



Here are the results for discharging:

Table 7: Discharging, $\hat{C}$; $R = 1\,\mathbf{M\Omega}$, $C = 31\,\mathbf{nF}$

Trial	$\hat{C}/\mathbf{nF}$	$\text{Error}/\mathbf{nF}$
1	31.43	±0.43
2	31.39	±0.42
3	32.02	±0.43



Strangely, unlike for the parallel case, the measurements from the experiment yielded values mostly similar to the value measured by the multimeter. This is further evidence that the results of the parallel case were likely a fluke, and that a redo of the experiment may yield better results.

3 Conclusions

Resistor capacitor circuits are time-dependent. The voltage over a capacitor being charged or discharged through a resisted current can become predictable over time with the right conditions. It has also been demonstrated that information about a resistor capacitor circuit can be deduced when otherwise unknown. Specifically, a resistor-capacitor circuit can be used to determine the capacitance of a capacitor when only a voltage oscilloscope is present.